DOCUMENT RESUME

ED 097 374 TM 004 012

AUTHOR Phillips, E. Ray
TITLE Development of Optimal Instructional Sequences.
PUB DATE [Apr 74]
NOTE 20p.; Paper presented at the Annual Meeting of the American Educational Research Association (59th, Chicago, Illinois, April 1974)

EDRS PRICE MF-$0.75 HC-$1.50 PLUS POSTAGE
DESCRIPTORS *Instructional Design; *Instructional Materials; *Sequential Learning; *Transfer of Training; *Validity
IDENTIFIERS *Learning Hierarchies

ABSTRACT

The report is a review of research results which concern sequencing (procedures for validating the hypothesized ordering of the subordinate tasks in a learning hierarchy) and practical applications of learning hierarchies in instructional design. Indirect versus direct validation procedures are examined in detail. (Author)

U S DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

BEST COPY AVAILABLE

ED 097374

DEVELOPMENT OF OPTIMAL INSTRUCTIONAL SEQUENCES

E. Ray Phillips

University of South Florida

An optimal instructional sequence can be defined as one which maximally facilitates achievement, transfer, and retention; requires the least amount of time for the learner to complete; and induces minimal anxiety and frustration on the learner's part (Phillips, 1971). Two questions arise logically from the formulation of the above definition. (1) Do optimal instructional sequences exist? (2) How are they determined and verified? The purpose of this paper is to review the empirical evidence supporting the feasibility of the development of optimal instruction sequences and to explore the problem of developing and validating such sequences.

Learning Hierarchies

A learning hierarchy may be characterized as "representing the most probable expectation of greatest positive transfer for an entire sample of learners whom we know nothing more than what specifically relevant skills they start with" (Gagne, 1968). Thus, it appears that learning hierarchy theory can serve as a foundation for determining optimal instructional sequences.

Paper presented at the Symposium, "Measurement Aspects of Instructional Sequencing", American Educational Research Association Annual Meeting, Chicago, April, 1974.

TM 004 012

Numerous studies support the hierarchial structure of learning theory. DiVesta and Walls (1967) demonstrated positive transfer from relevant "pre-utilization" training to the Maier two-string problem. Davis (1967) showed the effectiveness of transfer of previously learned verbal rules to switch-light problems, and a similar theme is developed by Overing and Travers (1966, 1967) in studies pertaining to the problem of hitting an underwater target. Scandura and Wells (1967) demonstrated positive transfer effects from prior learning in concrete situations involving relevant rules in problems concerning mathematical groups and combinatorial topology. Battig (1968) cited evidence that the learning of paired associates is typically facilitated by prior discrimination learning on stimulus-terms and response-terms, as well as by prior learning of stimulus coding responses. For a more extensive review of studies related to transfer of training, Schultz (1960) may be consulted.

Another type of study from which evidence about learning hierarchies may be derived is one which attempts to try out a total hierarchy in which the various levels of intellectual skills are to be learned in a single instructional sequence. Gagne (1963) hypothesized that "an individual will not be able to learn a particular topic if he has failed to achieve any of the subordinate topics that support it." This hypothesis was tested in several studies of the aforementioned type. Gagne and Paradise (1961) studied transfer relationships within a learning hierarchy for the task of solving linear equations. They concluded that when particular subordinate skills required for new learning were present in the learner high positive transfer resulted.

When they were absent, very low transfer took place. In a similar study, Gagne, Mayor, Garstens and Paradise (1962) measured the effects of learning program variations upon achievement. The results indicated that acquisition of subtasks at successively higher levels of the hierarchy was dependent upon prior mastery of subordinate subtasks. The authors also found that when the mediating effects were examined for subtasks intervening between higher and lower ones, proportions of achievement of higher subtasks indicated significant amounts of positive transfer. Several other studies (Gayne, 1962; Gagne, 1963; Gagne and Staff, University of Maryland Mathematics Project, 1965) yielded the same results reinforcing those of the previously discussed studies. In general, the results of these studies indicate that new skills and knowledge emerge from lower order knowledge, and that there is a significant amount of positive transfer from each successive subordinate level to the next higher level in a hierarchical ordering of such levels.

Sequencing Instructional Materials

Instructional design requires decisions about structuring content and designing and ordering a set of instructional tasks. Gagne (1967) and Briggs (1968) have proposed the use of instructional sequences that require the learner to follow a specific route through a content structure, suggesting that optimal instructional sequences can be developed by sequencing materials according to learning hierarchies.

Substantial evidence suggest that optimal learning sequences do, in fact, exist. Recent studies of sequencing (Brown, 1970; Niedermeyer, Brown, & Sulzen, 1969) indicated that Ss using materials sequenced according

to learning hierarchies performed reliably better than Ss using materials whose sequence was scrambled, relative to time to complete the instructional program, to errors made on the program, and to performance on a criterion test of complex problem-solving skills. Brown (1970) concluded that when a sequence involves tasks that are complex, ordering of problem-solving behaviors is an important factor in learning, even for bright and relatively mature learners. In summarizing research on sequencing mathematical tasks, Miller (1969) concluded that mastery of individual subtasks in a hierarchy can be achieved in several ways, including learning from randomly ordered sequences, but that logical sequencing appeared best in terms of overall efficiency and effectiveness. Roe (1962, p. 409) stated that "careful sequencing of items has a significant effect on student performance, at least for programs of some length and complexity." King (1970) described the above as key studies utilizing programs based on hierarchies and well controlled learning situations avoiding methodological weaknesses.

Phillips and Kane (1973) conducted a study to investigate the effects of seven different sequences upon the overall efficiency of learning from programed mathematical materials. No sequence maximally facilitated achievement, retention, and transfer, and required less time to complete. However, based on the group means, students using materials sequenced via a hierarchy developed by the Walbesser - AAAS (1968) technique performed more efficiently than student using any of the other six sequences.

Several studies (Levin & Baker, 1963; Miller, 1965; Payne, Krathwohl, & Gordon, 1967; Roe, Case, & Roe, 1962) however, suggest that varying

sequences of instructional stimuli that have high interdependency does not make much difference in effectiveness of instruction. However, some of these studies are plagued with design problems (King, 1970). Before sequencing instructional materials for use in classrooms, the effects of sequence upon time to achieve the terminal behavior, achievement, transfer, and retention should be investigated.

Hierarchy Validation Techniques

Validating a learning hierarchy is not a simple undertaking. Many researchers (Ausubel, 1963; Bruner, 1964; Gagne, 1965; Glaser, 1964; and Suppes, 1966) have long recognized that sequence is a critical variable in learning. The learner begins with simple tasks and progresses to increasingly complex tasks. However, both Gagne (1968) and Pyatte (1969) have pointed out that determination of this hierarchical ordering of subtasks from simplest to most complex is a major problem.

Gagne and Paradise (1961) were pioneers in learning hierarchy validation. Their approach was direct validation based on learners' responses to a programmed learning sequence and criterion tests administered immediately after the instructional program to establish pass-fail patterns for each component of the learning hierarchy. Consider the simple two-level hierarchy in Figure 1.

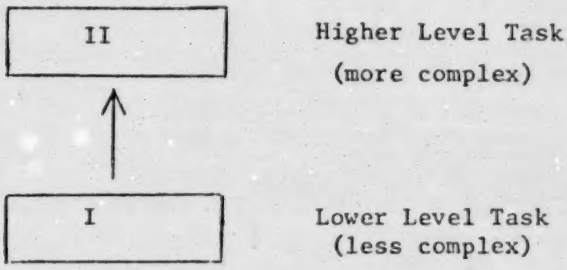

Figure 1. A Simple Two-level Hierarchy

Gagne's validation procedure was based on the assumption that task I must be mastered before task II can be mastered. Failure on task I would automatically produce failure on task II. Using + and - to represent pass and fail respectively, there are four possible pass-fail relationships which can be observed: (++), (+−), (−−), (−+). For example, the first relationship signifies that the learner passed (performed to criterion) both task I and task II. Only the relationship (+−) is in direct contradiction of the theory and indicates a flaw in ordering. The relationship (−+) (passed lower level task but failed higher level task) indicated a weakness in the instructional program but provides no information concerning the validity of the hierarchy.

To validate a hierarchy Gagne analyzed the pattern of responses of each transfer in the hierarchy. That is, he constructed a contingency table of the observed responses to a higher level task and the task immediately prerequisite to it as illustrated in Figure 1. He calculated the following ratio to determine the degree of validity of the hierarchy.

$$\text{Proportion of Positive Transfer (P+)} = \frac{(++) + (--)}{(++) + (--) + (+-)}$$

Perfect validity would be indicated by a ratio of 1.00. If all learners contradicted the theory, having observed patterns (+−), then the ratio would be zero. Thus, P+ is bounded above and below by 1 and 0 respectively. The degree of validity of any hierarchy is measured by P+ with the lower limit of acceptability for P+ being .90.

Phillips and Kane (1974) investigated the efficacy of this ratio when applied to test data alone. Using Gagne's task analysis a learning hier-

archy for the computational skills of whole number addition was constructed. Based on the hypothesized ordering of the subordinate levels, a test was constructed to assess mastery at each level. A second test utilizing a random ordering of the same items was constructed. Both tests were administered to a large sample of elementary school children in grades 3 through 6 in order to obtain a wide range of ability levels. The proportion of positive transfer between adjacent items on both tests was computed using Gagne's formula. The proportions between adjacent items on both tests were above .90, except in two instances. Thus both the hypothesized and the random hierarchies were validated by this procedure. The authors concluded that prior educational experiences confounded the issue of positive transfer when considering test data alone.

Walbesser (1969) has refined and extended Gagne's approach to hierarchy validation. Task analysis is used to generate hierarchies of subordintae subtasks. Learning sequences are designed to correspond to the hypothesized hierarchies. Pass-fail contingencies are used to test dependency of each individual task on its immediate prerequisite subtasks. The Staff for AAAS pointed out that a high proportion of positive transfer-Gagne's statistics - is a necessary but not a sufficient condition for a valid hierarchy. Using the pass-fail relationships defined by Gagne, the AAAS defined the following three ratios:

(1) Consistency ratio $= \frac{(++)}{(++) + (+-)}$

(2) Adequacy ratio $= \frac{(++)}{(++) + (-+)}$

(3) Completeness ratio $= \frac{(++)}{(++) + (--)}$

Ratio (1) is a measure of how consistent the data are with the hypothesized dependency. Ratio (2) is a measure of the adequacy of the identified subordinate tasks. Ratio (3) is a measure of the effectiveness of instructional materials designed to bring about learning. In the development of Science -- A Process Approach, The AAAS (1968) has considered high consistency, adequacy, and completeness ratios as the necessary and sufficient set of characteristics for a valid learning hierarchy. No significance test has been developed for either Gagne's ratio or those defined by the AAAS. Eisenberg and Walbesser (1971) have further refined this validation technique which includes the use of six different ratios.

Cox and Graham (1966) used the Guttman Scalogram Analysis (1944) to develop a sequencially scaled achievement test. Essentially, the Guttman technique (Torgerson, 1958) orders items such that from knowledge of a learner's total score, his response pattern to the set of items can be predicted. The coefficient of reproducibility which is given by one minus the ratio of the total number of errors to total number of responses

$$\text{Rep} = 1 - \frac{\text{total number of errors}}{\text{total number of responses}}$$

indicates the degree to which a set of items forms a perfect scale. Error is defined as instances where a subject passes a higher level item after failing a lower level prerequisite item. Guttman suggested .90 as an acceptable lower limit for Rep. Cox and Graham reported a reproducibility coefficient of .97 for their final arrangement of items and thus concluded their hierarchical ordering as valid.

Several investigators have attempted to use item difficulty as a means of validating a hierarchy. Studies by Stoker and Kropp (1964) and Herron (1965) showed that the cognitive skills in the Taxonomy (Bloom, 1956) did form a hierarchy. Items assessing skills higher up in the Taxonomy were more difficult than those at lower levels in the Taxonomy. Stoker, Kropp and Bashaw concluded that their results based on item difficulty validated the Taxonomy. Miller and Phillips (1974) used item difficulty in conjunction with the Walbesser technique to validate a hierarchy for the computational skills of rational number subtraction.

There are several methods of hierarchical analysis reported in the literature which are used in the generation of hierarchies rather than the validation of deductively analyzed hierarchies. McQuitty (1960) developed a procedure for determining if a hierarchical structure underlies a set of items. He began with a large item pool with no a priori assumptions regarding the relationship between items in terms of complexity. The procedure consisted of combining pairs of items or variables which correlated highest with each other to form new items. This procedure is repeated until one pair of items remains. When items are plotted on a linear scale and successive pairs are connected by lines the resulting diagram has a hierarchical structure. Smith (1968) employed McQuitty's method to investigate the hierarchical model underlying Bloom's Taxonomy. In general, Smith concluded that his analysis supported the Taxonomy rationale of a cumulative and hierarchical continuum of cognitive processes. McQuitty (1966) has further refined and improved his method of hierarchical syndrome analysis. Multiple Scalogram Analysis (MSA) developed by Lingoes (1963) can be used to generate a hierarchical ordering from a large set of items for which there are no a priori assumptions made regarding order.

The procedure essentially accomplished the same goal as the Guttman Technique with built in controls against spuriously high reproducibilities as a function of extreme marginal values. This procedure tends to produce several branches which have very little in common with one another. Resnick and Wang (1969) have used MSA to generate various hierarchies. The methods developed by McQuitty (1960) and Lingoes (1963), as well as several other methods of hierarchical analyses outlined by Torgerson (1958), are not readily adaptable for use in validating hypothesized hierarchies. In these methods the data must speak for themselves with no a priori assumptions concerning order. Carroll (Resnick and Wang, 1969) has developed a hierarchy validation procedure based on the correlation between items or subtasks. This method, like those of Gagne and Walbesser, is based on pass-fail contingency tables for all possible pairs of items in the hierarchy and the phi-phimax coefficients with a built-in control against artificial inflation due to extreme pass-fail rates. In discussing different approaches to hierarchy validation, Resnick and Wang (1969) cited no studies utilizing Carroll's method. Grib and Rimoldi (1960) developed a procedure for comparing two patterns of using the index of agreement. Listing each subject's responses to a set of items produces an observed matrix of responses with rows corresponding to items. An expected matrix can be formed based on an operational definition of what response patterns are expected from a given set of items. The only restriction on the expected matrix is that the subjects total score on the observed pattern. These two patterns can be compared and an index showing the amount of agreement between the two patterns can be computed.

Phillips and Kane (1973) conducted a study to investigate the efficacy of 7 procedures for ordering the subtasks in a learning sequence. An initial

hierarchy for rational number addition was constructed. A test to assess mastery at each of the 11 levels was administered to 163 elementary school children and the pass-fail relationships were analyzed using the following procedures: (1) Item difficulty (Nunnally, 1967), (2) The AAAS approach (AAAS Commission on Science Education, 1968), (3) The Guttman Technique (Torgerson, 1958), (4) Pattern analysis (Rimoldi and Grive, 1960), and (5) Correlation analysis (Phillips, 1971). For a detailed description of these procedures, see Phillips (1971). The subtasks were also sequenced according to the "usual" textbook sequence and randomly. To examine the adequacy of each validation procedure, programmed instructional materials were sequenced according to the hierarchy generated by each procedure. Subjects were randomly assigned to the 7 treatments and groups were compared on achievement, transfer, retention, and time to complete the instructional sequence. No sequence maximally facilitated achievement, retention, and transfer, and required less time to complete. However, based on group means, the AAAS procedure yielded the best sequence overall, and was judged the best hierarchy validation technique of the seven under investigations.

Conclusions

In reviewing the literature, one finds substantial evidence to support the general theory of the hierarchical structure of knowledge (studies cited previously). There seems to be little doubt that new skills and knowledge emerge from lower order knowledge, and that there is a significant amount of of positive transfer from each successive subordinate level to the next higher level in a hierarchical ordering of such levels. The studies on sequencing instructional materials reviewed suggest that the sequence of subordinate tasks in a learning hierarchy, after sufficient validation, should describe a teaching program that will effectively accomplish the instructional objectives. That is, an instructional sequence based on the levels in the hierarchy will represent an optimal route for acquisition of th terminal task by a sample of learners. Hence, it appears that the development of optimal instructional sequences becomes essentially a problem of validating a learning hierarchy.

The question of validating the ordering of the levels in a hierarchy is a complex and illusive problem. Gagne (1968) stated that various methods have been tried but none seem entirely satisfactory as yet. Several of these techniques were reviewed in this paper; and undoubtedly, others are being developed and tested. It seems imperative that efforts to develop more efficient procedures for validating learning hierarchies be continued.

Several questions have been raised regarding Gagne's method of empirical validation of the hypothesized ordering of a set of subordinate tasks (Walbesser, 1968; Phillips & Kane, 1974; Phillips, 1971; White, 1973). These include: small sample size, imprecisely defined use of task analysis, no

significance test for the proportion of positive transfer ratio, and errors of measurement. White (1973) has suggested two lines of further learning hierarchy research. (1) Use Gagne's model and try to minimize the above problems. (2) Use White's (1973) more rigorous model. White's method essentially states, for a set of k subtasks, teach them in all k! possible orderings to different groups. Based on the performance of the learners in each group, one determines the "best" ordering. This ordering represents a valid hierarchy. Obvious difficulties with this method, of course, are the large number of experimental groups that would be involved, the number of subjects needed, and the time involved in this type of validation. When the number of subtasks is large, White suggested that the research must resort back to Gagne's original design with some recommended changes. One of the more important of the recommendations is to replace indices such as Gagne's "proportion of positive transfer", the ratios proposed by Walbesser and Guttman's index of reproducibility by a statistical test of hierarchical dependence. White and Clark (1973) report the development of such a test.

Recommendations

Validating a learning hierarchy by either of White's suggested procedures is a tedious and costly undertaking. In either case, one must teach a sequence of tasks to a large sample of learners; and with the rigorous model, k! sequences must be presented to different groups of learners. Based upon the performance of the learners in the experimental groups, the best ordering is determined as the valid sequence. Thus, it seems that White would define a valid learning hierarchy similar to Kane, McDaniel and Phillips (1971). That is, a valid hierarchy is one which yields an optimal learning sequence.

Phillips (1971) labeled validation techniques that are based on test data alone are called indirect. It seems that the development of less expensive indirect validation techniques could greatly improve the use of learning hierarchies in designing instructional sequences.

A hypothetical case is considered below to illustrate how one might use an indirect procedure to validate a learning hierarchy. Suppose 20 subtasks are to be ordered hierarchically, and for simplicity's sake item difficulty (Nunnally, 1967) will be used as the basis of ordering (the author is not assuming nor proposing that item difficulty is an adequate validation procedure).

1. Construct a test with 3 items for each of the 20 subtasks. (Pass for each subtask will be correct responses to 2 of the 3 items for that subtask.)

2. Administer the test to a large sample of subjects to include a wide range of ability and achievement levels.

3. From the test results, calculate the item difficulty for each of the 20 items and order from highest to lowest. This will represent the "valid" ordering of the 20 subtasks.

Regardless of the validation technique used with the three steps above, one must have some evidence of the adequacy of the technique. That is, did the technique employed yield a valid hierarchy? Such evidence may be obtained in the following manner:

1. Design instructional materials for each of the 20 subtasks.

2. Sequence these materials according to the hierarchy generated by

the indirect procedure under scrutiny (in this case item difficulty.

3. Teach a large sample of learners selected at random using the above sequence of instructional materials.

4. Sequence the instructional material for each of the 20 subtasks in several other orderings generated by direct or indirect methods as well as sequence ordered from logical analysis of the content and sequences deliberately ordered in a manner which one believes would interfere with learning.

5. Randomly assign a large number of students to these groups.

6. At the completion of the instructional program compare each group on achievement, retention, transfer, attitude toward the program, and time needed to complete the program.

7. If the indirect procedure produces an optimal learning sequence, one concludes the procedure is adequate for generating a valid hierarchy. (Keep in mind more than one valid hierarchy could be generated from the same 20 subtasks).

Now that the procedure has been put to the ultimate test of actually sequencing instructional materials based on the hierarchy generated, one can apply the indirect procedure to a different set of subtasks with some confidence. That is, there's no need to rediscover the wheel each time a hierarchy is to be validated. The indirect procedure can be applied to test data alone without going through a costly instructional program to validate directly.

Hopefully new hierarchy validation technique will be developed which may be adaptable to the indirect approach to validating a learning hierarchy. With the development of adequate hierarchy validation techniques, the possibility of the development of optimal instructional sequences becomes a very real one.

BIBLIOGRAPHY

AAAS Commission on Science Education. Science - A process approach: An evaluation and its application, Second Report. American Association for the Advancement of Science, 1968.

Ausubel, D. P. The psychology of meaningful verbal learning. New York: Grune & Stratton, 1963.

Battig, W. F. Paired-associate learning. In T. R. Dixon & D. L. Horton (Eds.), Verbal behavior and general behavior theory. Englewood Cliffs, N. J.: Prentice-Hall, 1968.

Bloom, B. S. (Ed.) Taxonomy of educational objectives handbook I: New York: David McKay Company, Inc. 1956.

Briggs, L. J., & Angell, D. Programmed instruction in science and mathematics. Review of Educational Research, 1964, 34, 354-373.

Brown, J. L. Effects of logical and scrambled sequences in mathematical materials on learning with programmed instruction materials. Journal of Educational Psychology, 1970, 61, 41-45.

Bruner, J. S. Some theorems on instruction illustrated with reference to mathematics. In Theories of learning and instruction, Sixty-third Yearbook of the National Society for the Study of Education. Chicago: University of Chicago Press, 1964.

Cox, R. C., & Graham, G. T. The development of a sequentially scaled achievement test. Journal of Educational Measurement, 1966, 3, 147-150.

Davis, G. A. Detrimental effects of distraction, additional response alternatives, and longer response chains in solving switch-light problems. Journal of Experimental Psychology, 1967, 73, 45-55.

DiVesta, F. J., & Walls, R. T. Transfer of object-function in problem solving. American Educational Research Journal, 1967, 4, 207-216.

Eisenberg, T. A. & Walhesser, H. H. Learning hierarchies - Numerical considerations. Journal for Research in Mathematics Education, 1971, 2, 244-256.

Gagne, R. M. & Paradise, N. E. Abilies and learning sets in knowledge acquisition. Psychological Monographs, 1961, 75 (518).

Gagne, R. M., Major, J. R., Garstens, H. L., & Paradise, N. E. Factors in qcquiring knowledge of a mathematical task. Psychological Monographs, 1962, 76 (526).

Gagne, R. M. & Brown, L. T. Some factors in the programming of conceptual learning. Journal of Experimental Psychology, 1961, 62, 313-321.

Gagne, R. M. The acquisition of knowledge. Psychological Review, 1962, 69, 355-365.

Gagne, R. M. Learning and proficiency in mathematics. The Mathematics Teacher, 1963, 56, 620-626.

Gagne, R. M. The conditions of learning. New York: Holt, Rinehart & Winston, Inc., 1965.

Gagne, R. M. & Staff, University of Maryland Mathematics Project. Some factors in learning non-metric geometry. Monographs of the Society for Research in Child Development, 1965, 30, 42-49.

Gagne, R. M. Curriculum research and the promotion of learning. In R. E. Stake (Ed.), Perspectives of curriculum evaluation. AERA Monograph Series on Curriculum Evaluation, No. 1. Chicago: Rand McNally, 1967.

Gagne, R. M. Training and principles of learning. In E. A. Fleishman (Ed.), Studies in personnel and industrial psychology. Homewood, Ill.: The Dorsey Press, 1967.

Gagne, R. M. Learning hierarchies. Presidential address, Division 15, American Psychological Association, 1968.

Glaser, R. Implications of training research for education. In Theories of learning and instruction, Sixty-third Yearbook of the National Society for the Study of Education. Chicago: University of Chicago Press, 1964.

Guttman, L. A basis for scaling qualitative idea. American Sociological Review, 1944, 9, 139-150.

Herron, J. D. A factor analytic and statistical comparison of CHEM study and conventional chemistry in terms of their development of cognitive abilities. (Doctoral dissertation, Florida State University) Ann Arbor, Michigan: University Microfilms, 1965, No. 65-15, 466.

Kane, R. B., McDaniel, E. D. & Phillips, E. R. Analyzing learning hierarchies relative to transfer relationships within arithmetic. USOE Project No. 9-E-128, July, 1971, Purdue University. (Available from ERIC, No. ED 053965).

King, B. W. On scrambling instructional stimuli. Journal for Research in Mathematics Education, 1970, 1, 233-240.

Levin, G. R. & Baker, B. L. Item scrambling in a self-instructional program. Journal of Educational Psychology, 1963, 54, 138-143.

Lingoes, J. C. Multiple scalogram analysis: A set-theoretic model for analyzing dichotomous items. Educational and Psychological Measurement, 1963, 23, 501-524.

McQuitty, L. L. Hierarchical syndrome analysis. Educational and Psychological Measurement, 1960, 20, 293-304.

McQuitty, L. L. Improved hierarchical syndrome analysis of discrete and continuous data. Educational and Psychological Measurement, 1966, 26, 577-582.

Miller, H. R. An investigation into sequencing and prior information variables in a programed instruction unit for junior high school mathematics. Paper read at the DAVI Convention, Milwaukee, 1965.

Miller, P. L. & Phillips, E. R. Development of a learning hierarchy for the computational skills of rational number subtraction. Paper presented at The American Education Research Association Annual meeting, Chicago, 1974.

Niedermeyer, F. C., Brown, J., & Sulzen, R. Learning and varying sequences of ninth-grade mathematics materials. Journal of Experimental Education, 1969, 37, 61-66.

Nunnally, J. C. Psychometric theory. New York: McGraw-Hill, 1967.

Overing, R. L. R. & Travers, R. M. W. Effect upon transfer of variations in training conditions. Journal of Educational Psychology, 1966, 57, 179-188.

Overing, R. L. R. & Travers, R. M. W. Variation in the amount of irrelevant cues in training and test conditions and the effects upon transfer. Journal of Educational Psychology, 1967, 58, 62-68.

Payne, D. A., Krathwohl, D. R. & Gordan, J. The effect of sequence on programed instruction. American Educational Research Journal, 1967, 4, 125-132.

Phillips, E. R. Validating learning hierarchies for sequencing mathematical tasks. (Doctoral dissertation, Purdue University) Ann Arbor, Michigan: University Microfilms, 1971, No. 72-8008, 171.

Phillips, E. R. & Kane, R. B. Validating learning hierarchies for sequencing mathematical tasks in elementary school mathematics. Journal for Research in Mathematics Education, 1973, 4, 141-151.

Phillips, E. R. & Kane, R. B. Validating learning hierarchies. Florida Journal of Educational Research, 1974, (In press).

Pyatte, J. A. Some effects of unit structure of achievement and transfer. American Educational Research Journal, 1969, 6, 241-61.

Resnick, L. E., & Wang, M. C. Approaches to the validation of learning hierarchies. Paper read at 18th annual Western Regional Conference on Testing Problems. San Francisco, California, 1969.

Rimoldi, H. J. A. & Grib, T. F. Pattern analysis. The British Journal of Statistical Psychology, 1960, 13, 137-149.

Roe, A. A comparison of branching methods for programed learning Journal of Educational Research, 1962, 55, 407-416.

Roe, K. V., Case, H. W. & Row, A. Scrambled versus ordered sequence in autoinstructional programs. Journal of Educational Psychology, 1962, 53, 101-104.

Scandura, J. M. & Wells, J. N. Advance organizers in learning abstract mathematics. American Educational Research Journal, 1967, 4, 295-301.

Schultz, R. W. Problem solving behavior and transfer. Harvard Educational Review, 1960, 30, 61-67.

Smith, R. B. An empirical examination of the assumptions underlying the taxonomy of educational objectives: Cognitive domain. Journal of Educational Measurement, 1968, 5, 125-128.

Stoker, H. W. & Kropp, R. P. Measurment of cognitive processes. Journal of Educational Measurement, 1964, 1, 39-42.

Suppes, P. Mathematical concepts formation in children. American Psychologist, 1966, 21, 139-150.

Torgerson, W. S. Theory and methods of scaling. New York: John Wiley & Sons, Inc., 1958.

Walbesser, H. H. An evaluation model and its application. Washington, D.C.: American Association for The Advancement of Science, 1968.

White, R. T. Research into learning hierarchies. Review of Educational Research, 1973, 43, 361-375.

White, R. T. & Clark, R. M. A test of inclusion which allows for errors of measurement. Psychonetrika, 1973, 38, 77-86.